7.A.2
7.B.1

Nuclear Physics B13 (1969) 587-594. North-Holland Publ. Comp., Amsterdam

A MODEL FOR NON-DIFFRACTIVE CURRENT-CURRENT SCATTERING AMPLITUDES

M. BANDER *

Department of Nuclear Physics. Weizmann Institute of Science. Rehovoth, Israel

Received 20 May 1969

Abstract: A model for the amplitude obeying the current algebra sum rule is constructed. It satisfies many physical conditions required of this amplitude as analyticity, Regge asymptotics, decreasing form factors, etc. This amplitude likewise satisfies certain asymptotic sum rules and suggests the possible structure of the difference in elastic electron scattering from members of an SU(3) or SU(2) multiplet.

1. INTRODUCTION

Current algebra [1] yields a sum rule for one of the amplitudes occurring in current-hadron scattering. A model for this amplitude satisfying many physical conditions required ot it is presented. It is a generalization of the Veneziano [2] model to amplitudes involving two currents and hence having asymptotic properties differing from the usual Regge ones.

For simplicity we shall examine the scattering of U(3) × U(3) currents on spin-zero hadron states. We consider the scattering of a current of momentum q_1 on a state of momentum p_1 to a current of momentum q_2 and a state of momentum p_2:

$$p_1 + q_1 \to p_2 + q_2 \tag{1.1}$$

and introduce the following variables

$$\begin{aligned} p &= p_1 + p_2 , \\ \nu &= p \cdot (q_1 + q_2) , \\ t &= (p_1 - p_2)^2 , \\ s &= \tfrac{1}{2} (\nu + m_1^2 + m_2^2 + q_1^2 + q_2^2 - t) . \end{aligned} \tag{1.2}$$

* Alfred P. Sloan Foundation Fellow on leave from Department of Physics, University of California, Irvine, California.

The scattering amplitude is

$$M^{\alpha\beta}_{\mu\nu}(P,q_1,q_2) = -i(2\pi)^3 \sqrt{4\omega_1\omega_2} \times \int d^4x\, e^{iq_1\cdot x}\, \theta(x_0) \langle p_2 | [j^\alpha_\mu(x), j^\beta_\nu(0)] | p_1\rangle \,. \quad (1.3)$$

The j are currents with the superscripts referring to U(3) × U(3) indices; $\omega_i = \sqrt{p_i^2 + m_i^2}$, the energy of the state. The amplitude $M_{\mu\nu}$ can be decomposed into covariant tensors

$$M^{\alpha\beta}_{\mu\nu}(P,q_1,q_2) = P_\mu P_\nu A^{\alpha,\beta}(\nu,t,q_1^2,q_2^2) + q_{1\mu} P_\nu B^{\alpha\beta}(\nu,t,q_1^2,q_2^2) + q_{2\mu} P_\nu C^{\alpha\beta}(\nu,t,q_1^2,q_2^2) + \dots \,. \quad (1.4)$$

We shall likewise need the form factor between the hadron states

$$(2\pi)^3 \sqrt{4\omega_1\omega_2} \langle p_2 | j^\gamma_\mu(0) | p_1\rangle = P_\mu F^\gamma(t) + \Delta_\mu G^\gamma(t) \,. \quad (1.5)$$

The current algebra commutation relation

$$[j^\alpha_0(x,0), j^\beta_0(0)] = if^{\alpha\beta\gamma}\, j^\gamma_0(x)\delta^3(x) \quad (1.6)$$

and certain assumption on the asymptotic behaviour of the invariant amplitudes yield

$$\lim_{\nu\to\infty} \nu A^{\alpha\beta}(\nu,t,q_1^2,q_2^2) = 2if^{\alpha\beta\gamma} F^\gamma(t) \,. \quad (1.7)$$

It is for this A-amplitude that we shall construct a model. However we shall first discuss some of the requirements that A must satisfy:

(i) Dispersion relations in s or ν: The amplitude A has singularities in s (and in the crossed variable u; $\nu = s - u$). As we shall be dealing with an amplitude which gets no contribution from the vacuum trajectory in the t-channel we may assume that in the s-channel it is dominated by resonances [3]. As in the Veneziano model these singularities will be approximated by an infinite set of zero width resonance poles.

(ii) Crossing: $A(\nu\ldots) = -A(-\nu\ldots)$.

(iii) Vector-meson dominance of form factors: The residue of poles in the direct channel is a product of two form factors for the respective currents. These form factors will satisfy unsubtracted dispersion relations and will consist of an infinite sum of poles at masses of vector mesons whose quantum numbers match those of the relevant current. The requirement of an infinite set of vector mesons is consistent with the Veneziano model or with any model possessing an infinite set of parallel linearly rising daughter trajectories. Either of the above models possesses vector mesons spaced by inverse multiples of the trajectory slope.

(iv) Regge behavior: The amplitude A corresponds to a t-channel helicity amplitude with two units of helicity flip. Thus if $\alpha_\ell(t)$ is the leading allowed trajectory in the t-channel, we would expect the asymptotic behavior of A to be $\nu^{\alpha_t(t)-2}$. The imaginary part of A is taken to have this behavior. However, in order for the sum rule to be satisfied the real part of A must possess a fixed pole at $J = 1$ with a residue proportional to $F(t)$ (refs. [4, 5]).

(v) Superconvergence of semi-weak and strong amplitudes. The residue of a pole in q_1^2 or q_2^2 cannot have the fixed poles discussed above as the left-hand side of eq. (1.7) would then develop singularities in these variables. One way of insuring this is requiring these residues to have normal Regge asymptotic behavior.

(vi) Current conservation: If one of the currents, say 1, is conserved then from eq. (1.4) we obtain

$$\tfrac{1}{2}(\nu + m_2^2 - m_1^2) A^{\alpha\beta} + q_1^2 B^{\alpha\beta} + q_1 \cdot q_2 \, C^{\alpha\beta} = f^{\alpha\beta\gamma} F^{\gamma} . \tag{1.8}$$

In the kinematical situation [5] $q_1^2 = 0$, $q_1 \cdot q_2 = 0$ (implying $t = q_2^2$) we have

$$(s - m_1^2) \operatorname{Im} A = 0 \qquad \text{or} \qquad \operatorname{Im} A = \pi F(t) \delta(s - m_1^2) . \tag{1.9}$$

In other words in this limit only the Born term contributes and

$$A = F(t)/(s - m_1^2) . \tag{1.10}$$

In the subsequent sections we shall develop a representation for A satisfying all the above conditions. Likewise some speculations on amplitudes appearing in inelastic electron scattering and related to A will be presented.

2. MATHEMATICAL PRELIMINARIES

The model for A amplitude is based on the properties of the following functions:

$$G_k(s, t, q_1^2, q_2^2) = \int_0^1 \mathrm{d}v \, v^{-1-\alpha_s(s)} (1-v)^{1-\alpha_t(t)}$$

$$\int_0^1 \mathrm{d}u_1 \, \mathrm{d}u_2 (1 - v u_1 u_2)^{\alpha_t - k} \, u_1^{-\alpha_1(q_1^2)} \, u_2^{-\alpha_2(q_2^2)} , \tag{2.1}$$

with $k \leq 3$; α_s, α_t are trajectory functions in the s- and t-channels respectively and $\alpha_i(q_i^2)$ in the trajectory function for the vector mesons coupling to current i. We assume all trajectory functions to be linear. Keeping just the first two factors in the integrand of eq. (2.1) and integrating over v yields a representation of the Euler B-function which occurs in the Veneziano model. The representation is valid for $\alpha_s < 0$, $\alpha_t < 2$, $\alpha_i < 1$ and may be extended to other values by analytic continuation.

Many of the properties of this function may be best studied after expanding the third factor in the integrand and performing the subsequent integrations

$$G_k(s,t,q_1^2,q_2^2) = \frac{\Gamma(2-\alpha_t)}{\Gamma(k-\alpha_t)} \sum_{n=0}^{\infty} \frac{\Gamma(k+n-\alpha_t)\Gamma(n-\alpha_s)}{n!\,\Gamma(2+n-\alpha_s-\alpha_t)(n+1-\alpha_1)(n+1-\alpha_2)} . \quad (2.2)$$

The above converges for all cases when the summand is defined, i.e. for non-integer α and $k \leqslant 3$.

(i) Asymptotic behavior in s: We consider s or equivalently $\alpha_s(s)$ going to infinity along any ray not coinciding with the positive real axis [6]. Each term in the sum of eq. (2.2) behaves as $(-\alpha_s)^{\alpha_t-2}$. However, as the sum is not uniformly convergent in this limit we cannot conclude that the whole sum has this behavior. We separate the summation into two parts: $n < M$ and $n \geqslant M$ where M is chosen such that for $n > M$ and $|\alpha_s - n|$ large the summand may be approximated as close as we wish by

$$n^{k-\alpha_t-3}(n-\alpha_s)^{\alpha_t-2} . \quad (2.3)$$

The summation for $n < M$ has the normal Regge asymptotic behavior $(-\alpha_s)^{\alpha_t-2}$ while the contribution for $n \geqslant M$ is to as good an approximation as we please given by

$$\int_M^{\infty} \mathrm{d}n\, n^{k-\alpha_t-3}(n-\alpha_s)^{\alpha_t-2} . \quad (2.4)$$

The asymptotic limit of the imaginary part of eq. (1.4) is

$$\frac{(-1)^k \sin \pi\alpha_t}{(2+\alpha_t-k)M^{4-k}} \left(\frac{|\alpha_s|}{M}\right)^{\alpha_t-2} \quad (2.5)$$

and thus the asymptotic behavior of $\mathrm{Im}\, G_k$ is $(\alpha_s)^{\alpha_t-2}$. The large α_s structure of $\mathrm{Re}\, G_k$ depends on whether $\alpha_t > k-2$ or $\alpha_t < k-2$. In the first case the real part follows that the imaginary while in the latter

$$\mathrm{Re}\, G_k \sim \frac{(3-k)!}{(-\alpha_s)^{4-k}} \frac{1}{(k-\alpha_t-1)(k-\alpha_t-2)} . \quad (2.6)$$

We note in passing that for $k=3$, $\alpha_t < 1$ we have the desired $1/\alpha_s$ behavior at large energy.

Now let us look at the singularities of G in the various energies and momenta. In the s-channel G develops a pole whenever α_s is a positive integer m with residue

$$\frac{\Gamma(2-\alpha_t)}{\Gamma(k-\alpha_t)} \sum_{n=0}^{m} \frac{\Gamma(k+n-\alpha_t)}{\Gamma(2+n-m-\alpha_t)} \frac{(-1)^{n-m}}{(m-n)!} \frac{1}{(n+1-\alpha_1)(n+1-\alpha_2)} , \quad (2.7)$$

which is a polynomial α_t whose q_1^2 and q_2^2 behavior is dominated by poles whenever α_1 or α_2 are positive integers corresponding to vector-meson masses.

Likewise in α_t we have singularities for α_t integer corresponding to t-channel exchanges. We note that $\mathrm{Im}\, G$ has no pole at $\alpha_t = 1$ which would, in the subsequent discussion, correspond to a sense-nonsense transition. The real part does however develop at pole at this point [4, 5].

The residue of a pole in either α_1 or α_2 is one term in the summation of eq. (2.2) and thus has normal Regge behavior in accordance with requirement (v) of sect. 1.

3. BASIC MODEL

Using the functions G_k and their properties derived in sect. 2 we give the representation for the A-amplitude. There always exists the ambiguity of non-leading terms and we shall exploit this guarantee condition (vi) of sect. 1 in the case current 1 is conserved.

$$A^{\alpha,\beta}(\nu, t, q_1^2, q_2^2) = \alpha_s'(0)(1-\alpha_1(0))$$

$$\times\{X^\alpha X^\beta[(\alpha_t - 2)G_3(s, t, q_1^2, q_2^2) - (\alpha_t - 1 - \alpha_1(0))G_2(s, t, q_1^2, q_2^2)] - X^\alpha X^\beta(s \to u)\} \; . \tag{3.1}$$

where the X are a representation of U(3) $\times$ U(3). It is straightforward to check that the sum rule (1.7) is indeed satisfied. The left-hand side of eq. (1.7) in the indicated limit is

$$2if^{\alpha\beta\gamma}\frac{1-\alpha\ (0)}{1-\alpha(t)} \tag{3.2}$$

and the residue of the first pole in s is

$$\frac{(1-\alpha_1(0))^2}{(1-\alpha_1(q_1^2))(1-\alpha_2(q_2^2))} \; . \tag{3.3}$$

Thus the form factor determined from either eqs. (3.2) or (3.3) is

$$F_i(t) = \frac{1-\alpha_1(0)}{1-\alpha_t(t)} \tag{3.4}$$

and the sum rule is satisfied*. Requirements (i) to (v) of sect. 1 may be directly verified using the properties of G_k listed in sect. 2.

The verification condition (vi) depends on one further physical requirement. One notes that if current 1 is conserved then $\alpha_2(x) = \alpha_t(x)$. With this condition on the trajectory functions and in the limit indicated in (vi), i.e. $q_1^2 = 0$, $q_2^2 = t$ the integrals appearing in eq. (3.1) may be performed in closed form and yield:

$$A^{\alpha,\beta} = \alpha_s'(0)\,[X^\alpha X^\beta \frac{1}{\alpha_s(s)}\frac{1-\alpha_1(0)}{1-\alpha_t(t)} - X^\beta X^\alpha(s \to u)] \tag{3.5}$$

* Eq. (3.4) has been obtained by Predazzi [10].

and we note that only the Born term contributes in accordance with the requirement of current conservation.

4. EXTENSIONS

Two shortcoming of the representation based on eq. (3.1) are: One, form factors decrease with only one power of momentum transfer while experimentally their decrease seems to be faster [7]; two, from eq. (2.2) we note that the residue of the nth pole in α_1, which corresponds to the scattering of the nth physical vector meson of type one, has a singularity in α_2 *only* at the nth pole. Thus the physical nth vector meson of type one scatters only to the nth vector meson of type two. In this section we shall indicate a mechanism for overcoming both difficulties. Basically it is effected by taking sums of terms similar to eq. (2.1). Ignoring the requirement of current conservation the leading term in a representation for A in the case where the large momentum behavior of currents α, β, γ is $1/(q_1^2)^{1+l_1}$, $1/(q_2^2)^{l_2+1}$ and $1/(t)^{1+l_t}$ respectively, is

$$A^{\alpha\beta} \sim \frac{\Gamma(3+l_1+l_2-\alpha_t)}{\Gamma(2+l_t-\alpha_t)} [X^\alpha X^\beta \int_0^1 \mathrm{d}v\, v^{-1-\alpha_s} (1-v)^{1-\alpha_t}$$

$$\times \int_0^1 \mathrm{d}u_1\, \mathrm{d}u_2 (1-v u_1 u_2)^{\alpha_t-3-l_1-l_2} u_1^{-\alpha_1} (1-u_1)^{l_1} u_2^{-\alpha_2} (1-u_2)^{l_2} - X^\beta X^\alpha (s \to u)] . \tag{4.1}$$

Following the same procedure as in the previous sections it is easy to show that this amplitude does satisfy the current-algebra sum rule with the form factor given by expressions as

$$F(t) = \frac{\Gamma(1-\alpha)\Gamma(1+l)}{\Gamma(2+l-\alpha)} , \tag{4.2}$$

which has the desired asymptotic fall off. Likewise this construction allows for inelastic scattering of on-shell vector mesons.

As the nature of the fast fall off of the experimentally measured form factors is not understood we shall not pursue much further attempts to get detailed fits with experiment.

5. ASYMPTOTIC SUM RULES

A relation for A in the limiting situation $q_1^2 = q_2^2$, $q_1^2 + q_2^2 \to \infty$, t fixed has been derived by Bjorken and this author [8]

$$\lim_{q^2 \to -\infty} q^2 \frac{\partial A}{\partial \nu} (\nu, t, q^2, q^2) \Big|_{\nu=0} = 0 . \tag{5.1}$$

In this limit $s \to -q^2$. Following the same procedure as in sect. 2 we may show that

$$\lim_{q^2 \to \infty} \alpha(q^2) \frac{\partial G_k}{\partial \nu}(\nu, t, q^2, q^2)\bigg|_{\nu=0} = -\frac{1}{(4-\alpha_t)(3-\alpha_t)} \frac{1}{[\alpha(q^2)]^{4-k}} \qquad (5.2)$$

and we see that this limiting sum rule is valid. It is likewise valid for the more general models presented in sect. 4.

6. INELASTIC ELECTRON SCATTERING

Inaleastic electron scattering is described by two form factors, W_1 and W_2 which are related to the absorptive part of an amplitude similar to eq. (1.3)

$$P_\mu P_\nu W_2 + \ldots + W_1 g_{\mu\nu} = (2\pi)^3 \sqrt{4\omega_1\omega_2} \int e^{iq \cdot x} \langle p | [J_\mu(x), J_\nu(0)] | p \rangle \quad (6.1)$$

with $J_\mu(x)$ being the ordinary electromagnetic current. For the difference in inelastic scattering from two members of an isotopic multiplet the particles which may be changed in the t-channel are $I=1$, $C=+1$ namely the A_2 meson trajectory. It is therefore tempting to assume that for the difference in scattering from members of a multiplet

$$W_2^{I_t=1}(\nu, q^2) \sim \mathrm{Im}[G_3(s, 0, q^2, q^2) + \frac{1}{\alpha_t(0)-2} G_2(s, 0, q^2, q^2)] \, , \qquad (6.2)$$

In the limit $\nu \to \infty$, $q^2 \to -\infty$, $-q^2/\nu = \rho$ fixed, Bjorken [9] obtained a theorem for $W_2(\nu, q^2)$

$$\lim_{q^2 \to -\infty, \rho = -q^2/\nu \text{ fixed}} W_2(\nu, q^2) \to \frac{1}{q^2} F(\rho) \, . \qquad (6.3)$$

This theorem is non-trivial as long as $F(\rho)$ is not identically zero or infinity. From eq. (6.2) we may evaluate $F(\rho)$; for $\rho > 1$ and $\alpha_t(0) > 0$

$$F(\rho) \sim \frac{1}{(\rho-1)^2} \int_1^\infty \frac{(\chi+)^{\alpha_t(0)-1}}{(1+2\chi/p-1)^3} d\chi \sim (\rho-1)(\rho+1)^{\alpha_t(0)-3} \, . \qquad (6.4)$$

For the case where the form factors fall off as $(1/q)^{l+1}$

$$F(\rho) \sim (\rho-1)^{2l+1}(\rho+1)^{\alpha_t(0)-3-2l} \, . \qquad (6.5)$$

We note that the theorem is satisfied in all cases. For $\rho \gg 1$, $F(\rho) \sim \sim \rho^{\alpha_t(0)-2}$ which for $\alpha_t(0) \sim \frac{1}{2}$ is $\rho^{-\frac{3}{2}}$. It is tempting to suggest that this is the behavior of $F(\rho)$ for the difference in inelastic electron scattering from protons and neutrons.

7. DISCUSSION

The previous models provide one mechanism for the build up of a fixed pole, whose residue is independent of the external current masses, by resonances whose form factors vanish with momentum transfers as rapidly as we wish. Qualitatively features of this mechanism have been known to many. As the momentum transfer increases contributions of low-mass resonances decrease; however, this decrease is compensated by the appearance in the excitation spectrum of new high-spin resonances. The excitation of these resonances occurs as a result of the coupling of these higher mass intermediate states to more and more vector mesons dominating the form factors. A qualitative confirmation of this model would be the following behavior of the difference in inelastic electron scattering from multiplets. As the four momentum squared of the virtual photon became more and more space-like the excitation spectrum would decrease at the low mass end and at the same time extend to higher masses.

The question whether the algebra of currents has a solution (in terms of discrete states) is still left open. To settle this one would have write analogous formulae for other external particles and note whether the form factors evaluated using the different expressions agree.

The author wishes to thank the Department of Nuclear Physics of the Weizmann Institute for their hospitality.

REFERENCES

[1] S. Adler and R. Dashen, Current algebras (W. A. Benjamin, New York, 1967).
[2] G. Veneziano, Nuovo Cimento 57 (1968) 190.
[3] H. Harari, Phys. Rev. Letters 20 (1968) 1395;
P. G. O. Freund, Phys. Rev. Letters 20 (1968) 235.
[4] J. Bronzan, I. Gerstein, B. Lee and F. Low, Phys. Rev. Letters 18 (1967) 32;
V. Singh, Phys. Rev. Letters 18 (1967) 36.
[5] J. Bronzan, I. Gerstein, B. Lee and F. Low, Phys. Rev. 157 (1967) 1448.
[6] R. Roskies, Phys. Rev. Letters 21 (1968) 1851.
[7] W. K. H. Panofsky, Proc. 14th Int. Conf. on high-energy physics, Vienna, 1968 (CERN, Scientific Information Service, Geneva, Switzerland) p. 23.
[8] M. Bander and J. Bjorken, Phys. Rev. 174 (1968) 1704.
[9] J. Bjorken, SLAC preprint.
[10] E. Predazzi, Weak interactions and topics in dispersion physics (W. A. Benjamin, Inc., New York, 1963).